International System of Units
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As of 20 May 2019, the seven base units of the International System of Units[1] (SI) are exactly defined by

$$
\begin{array}{lrl}
\text{time} & \text{second} \equiv \text{s} & \equiv 9192631770/\Delta\nu_{\text{Cs}}, \\
\text{length} & \text{metre} \equiv \text{m} & \equiv (c/299792458)(9192631770/\Delta\nu_{\text{Cs}}), \\
\text{mass} & \text{kilogram} \equiv \text{kg} & \equiv \big(h/6.62607015\times10^{-34}\big)(299792458/c)^2(\Delta\nu_{\text{Cs}}/9192631770), \\
\text{electric current} & \text{ampere} \equiv \text{A} & \equiv \big(e/1.602176634\times10^{-19}\big)(\Delta\nu_{\text{Cs}}/9192631770), \\
\text{thermodynamic temperature} & \text{kelvin} \equiv \text{K} & \equiv \big(1.380649\times10^{-23}/k\big)\big(h/6.62607015\times10^{-34}\big)(\Delta\nu_{\text{Cs}}/9192631770), \\
\text{amount of substance} & \text{mole} \equiv \text{mol} & \equiv 6.02214076\times10^{23}/N_{\text{A}}, \\
\text{luminous intensity in a given direction} & \text{candela} \equiv \text{cd} & \equiv (K_{\text{cd}}/683)\big(h/6.62607015\times10^{-34}\big)(\Delta\nu_{\text{Cs}}/9192631770)^2
\end{array}
\tag{1}
$$

in terms of the following seven constants, which may be determined by experiment with low relative uncertainties,

$$
\begin{aligned}
\Delta\nu_{\text{Cs}} &\equiv \text{unperturbed ground state hyperfine transition frequency of the caesium 133 atom}, \\
c &\equiv \text{speed of light in vacuum}, \\
h &\equiv \text{Planck's constant}, \\
e &\equiv \text{elementary charge}, \\
k &\equiv \text{Boltzmann's constant}, \\
N_{\text{A}} &\equiv \text{Avagadro's constant}, \\
K_{\text{cd}} &\equiv \text{luminous efficacy of monochromatic radiation of frequency } 540\times10^{12}\ 1/\text{s}.
\end{aligned}
\tag{2}
$$

An SI derived unit is a product of integer powers of SI base units; the following 22 SI derived units have special names:

$$
\begin{array}{lrll}
\text{plane and phase angle} & \text{radian} \equiv \text{rad} & \equiv 1, & \\
\text{solid angle} & \text{steradian} \equiv \text{sr} & \equiv 1, & \\
\text{frequency} & \text{hertz} \equiv \text{Hz} & \equiv 1/\text{s}, & \\
\text{activity referred to a radionuclide} & \text{becquerel} \equiv \text{Bq} & \equiv 1/\text{s}, & \\
\text{absorbed dose, kerma} & \text{gray} \equiv \text{Gy} & \equiv \text{m}^2/\text{s}^2 & \equiv \text{J/kg}, \\
\text{dose equivalent} & \text{sievert} \equiv \text{Sv} & \equiv \text{m}^2/\text{s}^2 & \equiv \text{J/kg}, \\
\text{force} & \text{newton} \equiv \text{N} & \equiv \text{kg m/s}^2, & \\
\text{pressure, stress} & \text{pascal} \equiv \text{Pa} & \equiv \text{kg}/\big(\text{m s}^2\big) & \equiv \text{N/m}^2, \\
\text{energy, work, amount of heat} & \text{joule} \equiv \text{J} & \equiv \text{kg m}^2/\text{s}^2 & \equiv \text{N m}, \\
\text{power, radiant flux} & \text{watt} \equiv \text{W} & \equiv \text{kg m}^2/\text{s}^3 & \equiv \text{J/s}, \\
\text{electric charge} & \text{coulomb} \equiv \text{C} & \equiv \text{A s}, & \\
\text{electric potential difference} & \text{volt} \equiv \text{V} & \equiv \text{kg m}^2/\big(\text{A s}^3\big) & \equiv \text{W/A}, \\
\text{capacitance} & \text{farad} \equiv \text{F} & \equiv \text{A}^2\text{s}^4/\big(\text{kg m}^2\big) & \equiv \text{C/V}, \\
\text{electric resistance} & \text{ohm} \equiv \Omega & \equiv \text{kg m}^2/\big(\text{A}^2\text{s}^3\big) & \equiv \text{V/A}, \\
\text{electric conductance} & \text{siemens} \equiv \text{S} & \equiv \text{A}^2\text{s}^3/\big(\text{kg m}^2\big) & \equiv \text{A/V}, \\
\text{magnetic flux} & \text{weber} \equiv \text{Wb} & \equiv \text{kg m}^2/\big(\text{A s}^2\big) & \equiv \text{V s}, \\
\text{magnetic flux density} & \text{tesla} \equiv \text{T} & \equiv \text{kg}/\big(\text{A s}^2\big) & \equiv \text{Wb/m}^2, \\
\text{inductance} & \text{henry} \equiv \text{H} & \equiv \text{kg m}^2/\big(\text{A}^2\text{s}^2\big) & \equiv \text{Wb/A}, \\
\text{Celsius temperature} & \text{degree Celsius} \equiv {}^\circ\text{C} & \equiv \text{K}, & \\
\text{catalytic activity} & \text{katal} \equiv \text{kat} & \equiv \text{mol/s}, & \\
\text{luminous flux} & \text{lumen} \equiv \text{lm} & \equiv \text{cd} & \equiv \text{cd sr}, \\
\text{illuminance} & \text{lux} \equiv \text{lx} & \equiv \text{cd/m}^2 & \equiv \text{lm/m}^2.
\end{array}
\tag{3}
$$

The following decimal multiples may be inseparably prefixed to SI units to define new SI units, except that they may not be prefixed to the SI base unit kilogram ≡ kg but may instead be prefixed to the SI unit gram $\equiv \text{g} \equiv 10^{-3}$ kg:

$$
\begin{array}{rrrrr}
\text{yotta} \equiv \text{Y} \equiv 10^{24}, & \text{tera} \equiv \text{T} \equiv 10^{12}, & \text{hecto} \equiv \text{h} \equiv 10^{2}, & \text{milli} \equiv \text{m} \equiv 10^{-3}, & \text{femto} \equiv \text{f} \equiv 10^{-15}, \\
\text{zetta} \equiv \text{Z} \equiv 10^{21}, & \text{giga} \equiv \text{G} \equiv 10^{9}, & \text{deca} \equiv \text{da} \equiv 10^{1}, & \text{micro} \equiv \mu \equiv 10^{-6}, & \text{atto} \equiv \text{a} \equiv 10^{-18}, \\
\text{exa} \equiv \text{E} \equiv 10^{18}, & \text{mega} \equiv \text{M} \equiv 10^{6}, & \text{deci} \equiv \text{d} \equiv 10^{-1}, & \text{nano} \equiv \text{n} \equiv 10^{-9}, & \text{zepto} \equiv \text{z} \equiv 10^{-21}, \\
\text{peta} \equiv \text{P} \equiv 10^{15}, & \text{kilo} \equiv \text{k} \equiv 10^{3}, & \text{centi} \equiv \text{c} \equiv 10^{-2}, & \text{pico} \equiv \text{p} \equiv 10^{-12}, & \text{yocto} \equiv \text{y} \equiv 10^{-24}.
\end{array}
\tag{4}
$$

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[1] *The International System of Units (SI)*, 9th edition 2019, Bureau International des Poids et Mesures, Pavillon de Breteuil, F-92312 Sèvres Cedex, France, ISBN 9789282222720, https://www.bipm.org/utils/common/pdf/si-brochure/SI-Brochure-9.pdf.

The following non-SI units are accepted for use with the SI:

$$\begin{aligned}
&\text{plane and phase angle} & \text{degree} &\equiv {}^\circ & &\equiv \pi/180, \\
&\text{plane and phase angle} & \text{minute} &\equiv {}' & &\equiv \pi/10800 \equiv \pi/[(180)(60)]\,, \\
&\text{plane and phase angle} & \text{second} &\equiv {}'' & &\equiv \pi/648000 \equiv \pi/[(180)(60)(60)]\,, \\
&\text{logarithmic ratio} & \text{neper} &\equiv \text{Np}, & &m\ \text{Np means} \quad m = \ln{(X/X_0)}, \\
&\text{logarithmic ratio} & \text{bel} &\equiv \text{B}, & &m\ \text{B means} \quad m = \log{(X/X_0)}, \\
&\text{logarithmic ratio} & \text{decibel} &\equiv \text{dB}, & &m\ \text{dB means}\ m/10 = \log{(X/X_0)}, \\
&\text{time} & \text{minute} &\equiv \text{min} & &\equiv 60\ \text{s}, \\
&\text{time} & \text{hour} &\equiv \text{h} & &\equiv 3600\ \text{s} \equiv (60)(60)\,\text{s}, \\
&\text{time} & \text{day} &\equiv \text{d} & &\equiv 86400\ \text{s} \equiv (60)(60)(24)\,\text{s}, \\
&\text{length} & \text{astronomical unit} &\equiv \text{au} & &\equiv 149597870700\ \text{m}, \\
&\text{area} & \text{hectare} &\equiv \text{ha} & &\equiv 10000\ \text{m}^2 \equiv \text{hm}^2, \\
&\text{volume} & \text{litre} &\equiv \text{l}, \text{L} & &\equiv \text{m}^3/1000 \equiv \text{dm}^3 \equiv 1000\ \text{cm}^3, \\
&\text{mass} & \text{tonne} &\equiv \text{t} & &\equiv 1000\ \text{kg}, \\
&\text{energy} & \text{electronvolt} &\equiv \text{eV} & &\equiv 1.602176634 \times 10^{-19}\ \text{kg m}^2/\text{s}^2 \equiv 1.602176634 \times 10^{-19}\ \text{J}.
\end{aligned} \tag{5}$$

The unified atomic mass unit (also known as the dalton ≡ Da),

$$\text{unified atomic mass unit} \equiv \text{u} \equiv \begin{array}{l}\text{one-twelfth the mass of a free, neutral carbon 12 atom} \\ \text{at rest in its (nuclear and electronic) ground state}\end{array}, \tag{6}$$

is a non-SI unit accepted for use with the SI, is not exactly defined in terms of SI units, and is determined by experiment,[2]

$$\begin{aligned}
\text{u} &= 1.66053906660(50) \times 10^{-27}\ \text{kg}, & \text{u} &= 931.49410242(28)\ \text{MeV}/c^2, \\
\text{kg} &= 6.0221407621(18) \times 10^{26}\ \text{u}, & \text{MeV}/c^2 &= \quad 0.00107354410233(32)\ \text{u}.
\end{aligned} \tag{7}$$

The SI defining constants of Eq. (2) are exactly expressed in terms of the SI base units as

$$\begin{aligned}
\Delta\nu_{\text{Cs}} &\equiv 9192631770\ \ 1/\text{s} \equiv 9192631770\ \text{Hz}, \\
c &\equiv 299792458\ \text{m/s}, \\
h &\equiv 6.62607015 \times 10^{-34}\ \text{kg m}^2/\text{s} \equiv 6.62607015 \times 10^{-34}\ \text{J s} \\
&\equiv 6.62607015 \times 10^{-34}\ \text{J s}/\big(1.602176634 \times 10^{-19}\ \text{J/eV}\big) \equiv 4.1356676969238586461\ldots \times 10^{-15}\ \text{eV s}, \\
e &\equiv 1.602176634 \times 10^{-19}\ \text{A s} \equiv 1.602176634 \times 10^{-19}\ \text{C}, \\
k &\equiv 1.380649 \times 10^{-23}\ \text{kg m}^2/\big(\text{K s}^2\big) \equiv 1.380649 \times 10^{-23}\ \text{J/K} \\
&\equiv \big(1.380649 \times 10^{-23}\ \text{J/K}\big)/\big(1.602176634 \times 10^{-19}\ \text{J/eV}\big) \equiv 8.6173332621451774336\ldots \times 10^{-5}\ \text{eV/K}, \\
N_{\text{A}} &\equiv 6.02214076 \times 10^{23}\ 1/\text{mol}, \\
K_{\text{cd}} &\equiv 683\ \text{cd s}^3/\big(\text{kg m}^2\big) \equiv 683\ \text{lm/W}.
\end{aligned} \tag{8}$$

The following constants are exactly defined as products of powers of SI defining constants and exact numerical factors:[3]

$$\begin{aligned}
\text{Planck's constant divided by } 2\pi \equiv \hbar &\equiv h/(2\pi) & &\equiv 1.0545718176461563912\ldots\times 10^{-34}\ \text{kg m}^2/\text{s} \\
& & &\equiv 6.5821195695090656980\ldots\times 10^{-16}\ \text{eV s}, \\
\text{first radiation constant} \equiv c_1 &\equiv 2\pi h c^2 & &\equiv 3.7417718521927580113\ldots\times 10^{-16}\ \text{kg m}^4/\text{s}^3, \\
\text{second radiation constant} \equiv c_2 &\equiv hc/k & &\equiv 1.4387768775039338021\ldots\times 10^{-2}\ \text{K m}, \\
\text{Wien wavelength displacement law constant} \equiv b &\equiv hc/(y_{\max} k) & &\equiv 2.8977719551851726614\ldots\times 10^{-3}\ \text{K m}, \\
\text{Wien frequency displacement law constant} \equiv b' &\equiv x_{\max} k/h & &\equiv 5.8789257576468249466\ldots\times 10^{10}\ 1/(\text{K s})\,, \\
\text{Stefan-Boltzmann constant} \equiv \sigma &\equiv 2\pi^5 k^4/\big(15 h^3 c^2\big) & &\equiv 5.6703744191844294539\ldots\times 10^{-8}\ \text{kg}/\big(\text{K}^4\text{s}^3\big)\,, \\
\text{molar gas constant} \equiv R &\equiv N_{\text{A}} k & &\equiv 8.31446261815324\ \text{kg m}^2/\big(\text{mol K s}^2\big)\,, \\
\text{Faraday constant} \equiv F &\equiv N_{\text{A}} e & &\equiv 9.64853321233100184 \times 10^4\ \text{A s/mol}, \\
\text{Josephson constant} \equiv K_{\text{J}} &\equiv 2e/h & &\equiv 4.8359784841698363244\ldots\times 10^{14}\ \text{A s}^2/\big(\text{kg m}^2\big), \\
\text{magnetic flux quantum} \equiv \Phi_0 &\equiv h/(2e) & &\equiv 2.0678338484619293230\ldots\times 10^{-15}\ \text{kg m}^2/\big(\text{A s}^2\big), \\
\text{von Klitzing constant} \equiv R_{\text{K}} &\equiv h/e^2 & &\equiv 2.5812807459304506660\ldots\times 10^{4}\ \text{kg m}^2/\big(\text{A}^2\text{s}^3\big), \\
\text{conductance quantum} \equiv G_0 &\equiv 2e^2/h & &\equiv 7.7480917298636506466\ldots\times 10^{-5}\ \text{A}^2\text{s}^3/\big(\text{kg m}^2\big).
\end{aligned} \tag{9}$$

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[2] These are 2018 CODATA recommended values (based on data through the end of 2018) from the table of Fundamental Physical Constants at https://physics.nist.gov/cuu/Constants/Table/allascii.txt. The numbers in parentheses are one standard deviation uncertainties in the last two digits.

[3] For a black body in equilibrium at absolute temperature T, the wavelength at which Planck's wavelength distribution is maximum is $\lambda_{\max} = b/T = hc/(y_{\max} kT)$, where $y_{\max} \equiv 4.9651142317442763036\ldots$ maximizes $y^5/(e^y - 1)$, and the frequency at which Planck's frequency distribution is maximum is $\nu_{\max} = b'T = x_{\max} kT/h$, where $x_{\max} \equiv 2.8214393721220788934\ldots$ maximizes $x^3/(e^x – 1)$. Note that $\lambda_{\max}\nu_{\max} = c x_{\max}/y_{\max} = 0.56825266054974313110\ldots c < c$.