Magnetic dipole moments of some spin-$\frac{1}{2}$ particles

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The 2018 CODATA recommended values[1,2] of Planck's constant divided by 2π, the Bohr magneton, the nuclear magneton, electron, proton, neutron, muon, helion, and triton magnetic dipole moments, their ratios with respect to the Bohr magneton and the nuclear magneton, and their g factors[3] are

$$\begin{aligned}
\hbar = h/(2\pi) \equiv{}& \;\; 1.0545718176461563812\ldots \times 10^{-34}\ \mathrm{J\ s} \equiv 6.5821195695090656356\ldots \times 10^{-16}\ \mathrm{eV\ s},\\
\mu_\mathrm{e} ={}& -9.2847647043(28) \times 10^{-24}\ \mathrm{J/T}, \qquad \mu_\mu = -4.49044830(10) \times 10^{-26}\ \mathrm{J/T},\\
\mu_\mathrm{p} ={}& \;\; 1.41060679736(60) \times 10^{-26}\ \mathrm{J/T}, \qquad \mu_\mathrm{h} = -1.074617532(13) \times 10^{-26}\ \mathrm{J/T},\\
\mu_\mathrm{n} ={}& -9.6623651(23) \times 10^{-27}\ \mathrm{J/T}, \qquad \mu_\mathrm{t} = \;\; 1.5046095202(30) \times 10^{-26}\ \mathrm{J/T},\\
\mu_\mathrm{B} = e\hbar/(2m_\mathrm{e}) ={}& \;\; 9.2740100783(28) \times 10^{-24}\ \mathrm{J/T} = 5.7883818060(17) \times 10^{-5}\ \mathrm{eV/T},\\
\mu_\mathrm{e}/\mu_\mathrm{B} ={}& -1.00115965218128(18), \qquad \mu_\mu/\mu_\mathrm{B} = -4.84197047(11) \times 10^{-3},\\
\mu_\mathrm{p}/\mu_\mathrm{B} ={}& \;\; 1.52103220230(46) \times 10^{-3}, \qquad \mu_\mathrm{h}/\mu_\mathrm{B} = -1.158740958(14) \times 10^{-3},\\
\mu_\mathrm{n}/\mu_\mathrm{B} ={}& -1.04187563(25) \times 10^{-3}, \qquad \mu_\mathrm{t}/\mu_\mathrm{B} = \;\; 1.6223936651(32) \times 10^{-3},\\
\mu_\mathrm{N} = e\hbar/(2m_\mathrm{p}) ={}& \;\; 5.0507837461(15) \times 10^{-27}\ \mathrm{J/T} = 3.15245125844(96) \times 10^{-8}\ \mathrm{eV/T},\\
\mu_\mathrm{e}/\mu_\mathrm{N} ={}& -1.83828197188(11) \times 10^{3}, \qquad \mu_\mu/\mu_\mathrm{N} = -8.89059703(20),\\
\mu_\mathrm{p}/\mu_\mathrm{N} ={}& \;\; 2.79284734463(82), \qquad \mu_\mathrm{h}/\mu_\mathrm{N} = -2.127625307(25),\\
\mu_\mathrm{n}/\mu_\mathrm{N} ={}& -1.91304273(45), \qquad \mu_\mathrm{t}/\mu_\mathrm{N} = \;\; 2.9789624656(59),\\
g_\mathrm{e} = 4m_\mathrm{e}\mu_\mathrm{e}/(e\hbar) ={}& -2.00231930436256(35), \qquad g_\mu = 4m_\mu\mu_\mu/(e\hbar) = -2.0023318418(13),\\
g_\mathrm{p} = 4m_\mathrm{p}\mu_\mathrm{p}/(e\hbar) ={}& \;\; 5.5856946893(16), \qquad g_\mathrm{h} = 4m_\mathrm{p}\mu_\mathrm{h}/(e\hbar) = -4.255250615(50),\\
g_\mathrm{n} = 4m_\mathrm{p}\mu_\mathrm{n}/(e\hbar) ={}& -3.82608545(90), \qquad g_\mathrm{t} = 4m_\mathrm{p}\mu_\mathrm{t}/(e\hbar) = \;\; 5.957924931(12).
\end{aligned} \tag{1}$$

Electrons, protons, neutrons, muons, helions, and tritons are spin-$\frac{1}{2}$ particles. Their intrinsic angular momentum (spin) operators $\mathbf{S}$ are vector operators whose component operators $\mathbf{S}\cdot\mathbf{e}$, along every direction $\mathbf{e}$,[4] only have the eigenvalues $\hbar/2$ and $-\hbar/2$.[5] Their magnetic dipole moment operators are vector operators proportional to their spin operators but with different constants of proportionality, including different signs,

$$\begin{aligned}
\boldsymbol{\mu}_\mathrm{e} &= \mu_\mathrm{e} 2\mathbf{S}/\hbar, \qquad & \boldsymbol{\mu}_\mu &= \mu_\mu 2\mathbf{S}/\hbar,\\
\boldsymbol{\mu}_\mathrm{p} &= \mu_\mathrm{p} 2\mathbf{S}/\hbar, \qquad & \boldsymbol{\mu}_\mathrm{h} &= \mu_\mathrm{h} 2\mathbf{S}/\hbar,\\
\boldsymbol{\mu}_\mathrm{n} &= \mu_\mathrm{n} 2\mathbf{S}/\hbar, \qquad & \boldsymbol{\mu}_\mathrm{t} &= \mu_\mathrm{t} 2\mathbf{S}/\hbar.
\end{aligned} \tag{2}$$

The energy operator of such a magnetic dipole moment in a magnetic field $\mathbf{B} = B\mathbf{e}$ is

$$-\boldsymbol{\mu}\cdot\mathbf{B} = -B\boldsymbol{\mu}\cdot\mathbf{e} = -\mu B 2\mathbf{S}\cdot\mathbf{e}/\hbar \tag{3}$$

and, since the eigenvalues of $\mathbf{S}\cdot\mathbf{e}$ are $\hbar/2$ and $-\hbar/2$, the respective eigenvalues of $-\boldsymbol{\mu}\cdot\mathbf{B}$ are $-\mu B$ and μB; the respective eigenvalues are $[9.2847647043(28) \times 10^{-24}\ \mathrm{J/T}]B$ and $-[9.2847647043(28) \times 10^{-24}\ \mathrm{J/T}]B$ for an electron, $-[1.41060679736(60) \times 10^{-26}\ \mathrm{J/T}]B$ and $[1.41060679736(60) \times 10^{-26}\ \mathrm{J/T}]B$ for a proton, etc..

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[1] Except for the values of $\hbar$, these values are from the table of Fundamental Physical Constants at https://physics.nist.gov/cuu/Constants/Table/allascii.txt and are based on data through the end of 2018. Ten additional digits are given for the values of $\hbar$; the exact definitions $h \equiv 6.62607015 \times 10^{-34}$ J s and eV $\equiv 1.602176634 \times 10^{-19}$ J, which took effect on 20 May 2019, may be used to compute further digits.

[2] Numbers in parentheses are one standard deviation uncertainties; for example, the electron magnetic dipole moment μ_e is $-9.2847647043 \times 10^{-24}$ J/T with uncertainty $0.0000000028 \times 10^{-24}$ J/T.

[3] g_n, g_h, and g_t use proton mass m_p, not neutron mass m_n, helion mass m_h, or triton mass m_t; the mass ratios are $m_\mathrm{n}/m_\mathrm{p} = 1.00137841931(49)$, $m_\mathrm{h}/m_\mathrm{p} = 2.99315267167(13)$, and $m_\mathrm{t}/m_\mathrm{p} = 2.99371703414(15)$.

[4] A direction vector $\mathbf{e}$ has unit length, $\sqrt{\mathbf{e}\cdot\mathbf{e}} = 1$.

[5] The eigenvalues $\hbar/2$ and $-\hbar/2$ of $\mathbf{S}\cdot\mathbf{e}$ do not involve the cosine of an angle between $\mathbf{S}$ and $\mathbf{e}$.