Stereographic projection and Einstein energy

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(Working notes, not for publication)

For the geometry shown in the figure,[1] x and ξ are given in terms of θ and each other by

$$x = 2R\tan\tfrac{\theta}{2} = -\frac{4R^2}{\xi}, \qquad \xi = -\frac{2R}{\tan\frac{\theta}{2}} = -\frac{4R^2}{x}, \tag{1}$$

infinitesimals dx, $d\xi$, and $d\theta$ are given in terms of each other by

$$dx = \frac{1}{\cos^2\frac{\theta}{2}}Rd\theta = \frac{4R^2}{\xi^2}d\xi, \qquad d\xi = \frac{1}{\sin^2\frac{\theta}{2}}Rd\theta = \frac{4R^2}{x^2}dx, \qquad Rd\theta = \frac{1}{1+\frac{x^2}{4R^2}}dx = \frac{1}{1+\frac{\xi^2}{4R^2}}d\xi, \tag{2}$$

derivatives with respect to x, ξ, and θ are given in terms of each other by

$$\frac{\partial}{\partial x} = \frac{\cos^2\frac{\theta}{2}}{R}\frac{\partial}{\partial\theta} = \frac{\xi^2}{4R^2}\frac{\partial}{\partial\xi}, \qquad \frac{\partial}{\partial\xi} = \frac{\sin^2\frac{\theta}{2}}{R}\frac{\partial}{\partial\theta} = \frac{x^2}{4R^2}\frac{\partial}{\partial x}, \qquad \frac{1}{R}\frac{\partial}{\partial\theta} = \left(1+\frac{x^2}{4R^2}\right)\frac{\partial}{\partial x} = \left(1+\frac{\xi^2}{4R^2}\right)\frac{\partial}{\partial\xi}, \tag{3}$$

and the sum of the derivatives with respect to x and ξ equals $\frac{1}{R}$ times the derivative with respect to θ,

$$\frac{1}{R}\frac{\partial}{\partial\theta} = \frac{\partial}{\partial x} + \frac{\partial}{\partial\xi}. \tag{4}$$

Multiplication of Eq. (4) by $-i\hbar c$, where $\hbar = \frac{h}{2\pi}$, h is Planck's constant, and c is the speed of light, and use of

$$R\theta = ct, \qquad x = cx_0, \qquad \xi = c\xi_0 \tag{5}$$

gives

$$-i\hbar\frac{\partial}{\partial t} = -i\frac{\hbar c}{R}\frac{\partial}{\partial\theta} = -i\hbar c\frac{\partial}{\partial x} - i\hbar c\frac{\partial}{\partial\xi} = -i\hbar\frac{\partial}{\partial x_0} - i\hbar\frac{\partial}{\partial\xi_0}. \tag{6}$$

This equation equates Einstein energy to the sum of Minkowski energy and an approximate gravitational energy for light that travels on a great circle of the universe, where R is the radius of the universe,[2] t is Einstein time, x_0 is Minkowski time (in a fictitious flat Minkowski spacetime tangent to the universe at a point of observation), ξ_0 is a conformally inverted time (related to a fictitious flat Minkowski spacetime tangent to the universe at the antipode to the point of observation), the differential operator $-i\hbar\frac{\partial}{\partial t}$ represents the Einstein energy operator which governs time evolution when it acts on wavefunctions parameterized by Einstein time, $-i\hbar\frac{\partial}{\partial x_0}$ represents a Minkowski energy operator, and $-i\hbar\frac{\partial}{\partial\xi_0}$ represents an approximate gravitational energy operator.

[1] "Maxwell's Influence on Geometry", Irving Segal, in *J. C. Maxwell, the sesquicentenial symposium: new vistas in mathematics, science, and technology*, Edited by M. S. Berger, North-Holland, Amsterdam (1984), pages 245-262, especially pages 256-259.

[2] "Apparent superluminal sources, comparative cosmology and the cosmic distance scale", I. E. Segal, *Monthly Notices of the Royal Astronomical Society* **242**, 423-427 (1990), gives the estimate $R \simeq 160 \pm 40$ million parsecs. My fit to the same observed data gives $R \simeq 154.7$ million parsecs $\simeq 504.6$ million lightyears.